

Rational Root Theorem

$$\begin{aligned} \text{Expand } f(x) &= (2x+1)(2x-1)(3x+2) \\ &= (4x^2 - 1)(3x+2) \\ &= 12x^3 + 8x^2 - 3x - 2 \end{aligned}$$

Now factor $P(x) = 12x^3 + 8x^2 - 3x - 2$

Poss. roots: $\pm 1, \pm 2$
 $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Try $x = \frac{1}{2}$ $12(\frac{1}{2})^3 + 8(\frac{1}{2})^2 - 3(\frac{1}{2}) - 2 = 0$

$(2x-1)$ is a factor $2(x - \frac{1}{2}) = 2x-1$

$$\begin{array}{r} \frac{1}{2} \begin{array}{|c|c|c|c|} \hline 12 & 8 & -3 & -2 \\ \hline \downarrow & 6 & 7 & 2 \\ \hline \end{array} \\ \hline x \cdot \frac{1}{2} \begin{array}{|c|c|c|c|} \hline 12 & 14 & 4 & 0 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \frac{12x^2 + 14x + 4}{2} \quad \rightarrow \quad 6x^2 + 7x + 2 \\ \frac{2x-1}{2} \overline{) 12x^3 + 8x^2 - 3x - 2} \\ \underline{-(12x^3 - 6x^2)} \quad \downarrow \\ 0 \quad 14x^2 - 3x \\ \underline{-(14x^2 - 7x)} \quad \downarrow \\ 0 \quad 4x - 2 \\ \underline{-(4x - 2)} \\ 0 \end{array}$$

$$\begin{aligned} &(2x-1)(6x^2+7x+2) \quad \begin{matrix} x:12 \\ x:7 \end{matrix} \\ &(2x-1)(6x^2+4x+3x+2) \\ &(2x-1)(2x(3x+2)+1(3x+2)) \\ &(2x-1)(2x+1)(3x+2) \end{aligned}$$

$2x-1 \rightarrow \frac{1}{2}$
 $2(x - \frac{1}{2})$